

THE DRELL-HEARN-GERASIMOV SUM RULE<sup>1</sup>

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## ABSTRACT

The Drell-Hearn-Gerasimov (DHG) sum rule relates the helicity structure of the photoabsorption cross section to the anomalous magnetic moment of the nucleon. It is based on Lorentz and gauge invariance, crossing symmetry, causality and unitarity. A generalized DHG sum rule may be derived for virtual photons. At low momentum transfer this generalized sum rule is saturated by the resonance region, at high momentum transfer it may be expressed by the parton spin distributions measured in deep inelastic scattering. The longitudinal-transverse interference determines the Cottingham sum rule, which is related to the electric and magnetic form factors over the whole range of momentum transfer.

## KEYWORDS

Photoabsorption, sum rules, helicity, asymmetries, spin structure, quark model

## INTRODUCTION

The existence of internal degrees of freedom manifests itself in a finite size of the nucleon, described by a form factor of a Dirac current and an anomalous magnetic moment multiplied by the Pauli form factor. By the same token a spectrum of excited states appears, a series of resonances in the mass region of 1-2 *GeV* and a flat continuum at higher energies, logarithmically rising at the highest observed energies between 200 - 300 *GeV*. Finite size effects in the ground state and the existence of an excitation spectrum are not all independent phenomena, but closely intertwined by sum rules and low energy theorems (LET).

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On the experimental side, photo- and electronuclear reactions are a particularly clean instrument to investigate the resonance region and to analyze the multipole content of the individual resonance contributions. With the advent of electron accelerators of high current and large duty-factor, new classes of experiments including polarization degrees of freedom have become possible. Such investigations range from threshold production of mesons to detailed studies of the helicity structure in the resonance region. The helicity structure of the cross section is expected to change at momentum transfers of the order of the vector meson masses.

The Drell-Hearn-Gerasimov (DHG) and Burkhardt-Cottingham (BC) sum rules connect the helicity structure of the cross sections in the inelastic region with ground state properties. Being based on general principles of physics like Lorentz and gauge invariance, crossing symmetry, causality and unitarity, these sum rules are an important consistency check for our understanding of the hadronic structure. They have never been measured directly. However, an analysis of pion photoproduction indicates some problems with the proton-neutron difference for the DHG sum rule. New experiments are underway to investigate these questions. Of particular interest is the question whether and how fast these sum rules converge as functions of the excitation energy. A failure to converge would shed serious doubts on our present understanding of hadronic structure and send the model-builders back to the drawing board.

As function of momentum transfer  $Q^2$ , chiral perturbation theory (ChPT) predicts the slope of the DHG integral at the real photon point. However, the loop expansion of ChPT breaks down in the region of the vector meson resonances, where the helicity structure changes abruptly. Similarly, we have solid predictions for  $Q^2 \rightarrow \infty$  from perturbative  $QCD$ . In the scaling region the DHG and BC sum rules may be directly expressed by the spin distribution functions of the quarks, the object of deep inelastic lepton scattering. Again, perturbative  $QCD$  breaks down if we approach the region of the vector meson masses, now from above. Corresponding to the pole structure in the complex plane, the resonance region will define a circle of convergence for both an expansion at the origin (the loops of ChPT) and at infinity (higher twists of perturbative  $QCD$ ).

In the following sect. 2 we will discuss the "classical" DHG sum rule for real photons. The more general framework of electroproduction including polarization degrees of freedom will be outlined in sect. 3. Appropriately defined integrated cross sections yield a generalization of the DHG sum rule to virtual photons and, derived from the longitudinal-transverse interference, the BC sum rule. Theories and models for these sum rules will be presented in sect. 4. Finally, we will briefly review the existing information on the helicity structure of the low-lying resonances in sect. 5, and draw some conclusions in sect. 6.

## THE DHG SUM RULE FOR REAL PHOTONS

The differential cross section for Compton scattering off the nucleon (for the kinematics see Fig. 1) may be decomposed into the contribution of the point-like Dirac particle as evaluated by Klein and Nishina (1929), additional contributions of the anomalous magnetic moment  $\kappa$  as given by Powell (1949) and terms arising from virtual excitations,

e.g. the polarizabilities of the nucleon,

$$\frac{d\sigma}{d\Omega} = \left. \frac{d\sigma}{d\Omega} \right|_{KN} + \left. \frac{d\sigma}{d\Omega} \right|_P(\kappa, \kappa^2, \kappa^3, \kappa^4) + \left. \frac{d\sigma}{d\Omega} \right|_{pol}. \quad (1)$$

In the case of forward scattering (photon scattering angle  $\theta = 0$ ), only the terms of quartic order in  $\kappa$  remain finite. The corresponding scattering amplitude,

$$T(\omega, \theta = 0) = \hat{\epsilon}'^* \cdot \hat{\epsilon} f(\omega) + i \vec{\sigma} \cdot (\hat{\epsilon}'^* \times \hat{\epsilon}) g(\omega), \quad (2)$$

contains a spin-flip amplitude  $g$  and a no-flip amplitude  $f$ , both functions of the photon energy  $\omega$ . The polarization vectors of the initial and final photon are denoted by  $\epsilon$  and  $\epsilon'$ , respectively, and  $\vec{\sigma}$  is the spin of the nucleon.

The amplitudes  $f$  and  $g$  may be expanded into a power series in  $\omega$  whose leading terms are determined by low energy theorems (LET) based on relativity and gauge invariance (Low, 1954; Gell-Mann and Goldberger, 1954)

$$f(\omega) = -\frac{e^2}{m} + (\alpha + \beta)\omega^2 + [\omega^4], \quad (3)$$

$$g(\omega) = -\frac{e^2 \kappa^2}{2m^2} \omega + \gamma \omega^3 + [\omega^5]. \quad (4)$$

The leading term in  $f$  is the famous Thomson limit, the next order term is the contribution of the scalar polarizabilities of the nucleon, a sum of electric ( $\alpha$ ) and magnetic ( $\beta$ ) terms. The leading term in the spin-flip amplitude is proportional to the square of the anomalous magnetic moment; the next order term is the vector polarizability. The low energy limit of  $g$  is due to a Feynman graph with  $\kappa$  operating at both  $\gamma NN$  vertices, leading to the  $\kappa^4$  contribution in the total cross section.

The two terms  $f$  and  $g$  may be separated by an experiment using circularly polarized photons and nucleons polarized with spin parallel or antiparallel to the photon momentum. As has been shown in Fig. 2, the former situation leads to a state with overall spin  $J_z = \frac{3}{2}$ , the latter process to  $J_z = \frac{1}{2}$ . The corresponding amplitudes may be evaluated using eq. (2),

$$T_{3/2} = f - g, \quad T_{1/2} = f + g. \quad (5)$$

The optical theorem relates the imaginary parts of these amplitudes to the corresponding total absorption cross sections,

$$\text{Im } T_{1/2,3/2}(\omega) = \frac{\omega}{4\pi} \sigma_{1/2,3/2}(\omega). \quad (6)$$

Furthermore  $f$  is an even and  $g$  an odd function under  $\omega \rightarrow -\omega$  (crossing symmetry). On the basis of analyticity, unitarity and crossing symmetry, we may write a dispersion relation for  $g$ ,

$$\text{Re } g(\omega) = \frac{2\omega}{\pi} \int_{thr}^{\infty} \frac{d\omega'}{\omega'^2 - \omega^2} \frac{\omega'}{4\pi} \frac{\sigma_{1/2} - \sigma_{3/2}}{2}. \quad (7)$$

Since the threshold energy is of the order of the mass of the pion,  $m_\pi$ , this expression may be expanded into a power series in  $\omega$ . Comparing this series with the low energy

expansion, eq. (4), we obtain the DHG sum rule (Drell and Hearn, 1966; Gerasimov, 1966)

$$-\frac{\kappa^2}{4} = \frac{m^2}{8\pi^2\alpha} \int_{-\infty}^{\infty} \frac{d\nu}{\nu} (\sigma_{1/2}(\nu) - \sigma_{3/2}(\nu)) \stackrel{!}{=} I(Q^2 = 0). \quad (8)$$

Here and in the following we denote

$$\begin{aligned} \nu &= \frac{p \cdot q}{m} = \omega_{lab} \\ Q^2 &= -q^2 = \begin{cases} 0 & : \text{ real photons} \\ > 0 & : \text{ electron scattering,} \end{cases} \end{aligned} \quad (9)$$

and  $\alpha = e^2/4\pi \approx 1/137$ . On the *rhs* of eq. (8) we have defined the real photon point of a function  $I(Q^2)$  whose meaning will become clear in the following section. Similar to eq. (8) also the vector polarizability (or higher moments!) may be related to sum rules,

$$\gamma = \frac{1}{4\pi^2} \int \frac{d\nu}{\nu^3} (\sigma_{1/2} - \sigma_{3/2}). \quad (10)$$

In the more general formalism of photoabsorption (or electron scattering, see sect. 3), the helicity cross sections are related to the total transverse ( $\sigma_T$ ) and "transverse-transverse" ( $\sigma_{TT'}$ ) cross sections,

$$\sigma_T = \frac{\sigma_{3/2} + \sigma_{1/2}}{2} \quad (11)$$

$$\sigma_{TT'} = \frac{\sigma_{3/2} - \sigma_{1/2}}{2}. \quad (12)$$

Our experimental knowledge about these quantities is summarized in Figs. 3 and 4. The cross section  $\sigma_T$  clearly shows the first and second resonance region, indications of two more broad peaks and a nearly constant value to energies of about  $180\text{GeV}$ . The more recent DESY data lead up to the order of  $300\text{GeV}$  and show a slow logarithmic increase. As a consequence a dispersion relation for the Thomson term would not converge, and only a once-subtracted dispersion relation can be established for the sum of the scalar polarizabilities,  $\alpha + \beta$ . Being the difference of the helicity cross sections,  $\sigma_{TT'}$  is expected to decrease slowly with  $\nu$ , which would guarantee the convergence of the DHG integral, eq. (8).

The DHG sum rule has never been measured directly, the results shown in Fig.4 are essentially based on phase shift analyses of pion photoproduction using some estimates for the two-pion background. It involves data on both the proton and the neutron, because  $\kappa^2$  has the isospin dependence

$$\kappa^2 = (\kappa_S + \tau_0 \kappa_V)^2 = \kappa_V^2 + \kappa_S^2 + 2\kappa_S \kappa_V \tau_0. \quad (13)$$

Obviously the DHG integral  $I$  is dominated by the isovector moment  $\kappa_V$  (term  $I_{VV}$ ), the proton-neutron difference ( $I_{SV}$ ) is smaller by an order of magnitude and the contribution of the isoscalar moment ( $I_{SS}$ ) is practically negligible. The experimental data show clear indications for resonance structures with oscillating sign of the integrand of  $I$  (see Fig. 4). A more detailed multipole decomposition of  $I$  is given in table 1. It shows good agreement between experiment and the sum rule prediction for  $I_{VV}$ , but a large discrepancy for  $I_{SV}$ .

This has led to the speculation that the latter integral might need a subtraction. Chang *et al.* (1992) have tried to reconcile experiment and theory within the framework of a generalized current algebra. However, the paper has never been published.

Table 1: The multipole structure of the DHG integral  $I_p(Q^2 = 0)$  for the 3 isospin channels VV, SV and SS (see text). For the definitions of the resonances and multipoles see (Drechsel and Tiator, 1992). The "experimental" numbers are obtained by an analysis of pion photoproduction (Karliner, 1973).

resonance	multipole	$I^{VV}$	$I^{SV}$	$I^{SS}$
$P_{33}, 3/2^+$	$M_{1+}(E_{1+})$	-1.05	+.09	-
	( $\Delta_{1232}$ only)	-.93	)	
$S_{11}, 1/2^-$	$E_{0+}$	+.65	-.09	-
$P_{11}, 1/2^+$	$M_{1-}$	+.04	-.01	-
$D_{13}, 3/2^-$	$E_{2-}, M_{2-}$	-.26	-.05	-
$F_{15}, 5/2^+$	$E_{3-}, M_{3-}$	-.04	-.03	-
	$2\pi$ background	-.20	-.06	-
	experiment	-.86	-.15	small
	DHG	-.86	+.06	-.001

## THE GENERALIZED DHG FOR ELECTRON SCATTERING

The kinematics of lepton scattering with polarization degrees of freedom is shown in Fig. 5a for target polarization. The (longitudinal) polarization of the high energy electron is denoted by  $h = \vec{\sigma} \cdot \hat{k} \rightarrow \pm 1$ , the polarization  $\vec{P}$  of the target nucleon may be decomposed into a coordinate system with  $\hat{e}_z = \hat{q}$ , along the direction of the virtual photon,  $\hat{e}_x \perp \hat{e}_z$  in the electron scattering plane and in the hemisphere of the outgoing electron, and  $\hat{e}_y = \hat{e}_z \times \hat{e}_x$  perpendicular to the scattering plane. Note that in the standard EMC/SLAC experiment,  $\hat{P} = \pm \hat{k}$ , e.g. the spins of nucleon and electron are parallel or antiparallel.

In the case of a coincidence experiment, e.g.  $e + p \rightarrow e' + p' + \pi$ , the recoil polarization is usually analyzed in a coordinate system connected with the reaction plane of the  $p' - \pi$  system. Its axes are denoted by  $\hat{l}$  (along the direction of  $p'$ ),  $\hat{t}$  (transverse, in the reaction plane) and  $\hat{n}$  (normal to the reaction plane), as shown in Fig. 5b. The cross section for such an experiment is given by (Drechsel and Tiator, 1992)

$$\frac{d\sigma}{d\Omega_{e'} dk_{e'} d\Omega_{\pi}} = \Gamma \frac{d\sigma^{(v)}}{d\Omega_{\pi}}, \quad (14)$$

where  $\Gamma$  is the flux and  $d\sigma^{(v)}/d\Omega_{\pi}$  the differential cross section for the virtual photon,

$$\begin{aligned}
\frac{d\sigma^{(v)}}{d\Omega_\pi} &= \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \left\{ R_T + P_n R_T^n \right. \\
&\quad + \varepsilon_L (R_L + P_n R_L^n) \\
&\quad + \sqrt{2\varepsilon_L(1+\varepsilon)} \left( (R_{TL} + P_n R_{TL}^n) \cos \Phi + (P_l R_{TL}^l + P_t R_{TL}^t) \sin \Phi \right) \\
&\quad + \varepsilon \left( (R_{TT} + P_n R_{TT}^n) \cos 2\Phi + (P_l R_{TT}^l + P_t R_{TT}^t) \sin 2\Phi \right) \\
&\quad + h\sqrt{2\varepsilon_L(1-\varepsilon)} \left( (R_{TL'} + P_n R_{TL'}^n) \sin \Phi + (P_l R_{TL'}^l + P_t R_{TL'}^t) \cos \Phi \right) \\
&\quad \left. + h\sqrt{1-\varepsilon^2} (P_l R_{TT'}^l + P_t R_{TT'}^t) \right\}, \tag{15}
\end{aligned}$$

with  $\varepsilon$  and  $\varepsilon_L$  the transverse and "longitudinal" polarizations of the virtual photon,  $k_\gamma^{cm}$  the "photon equivalent energy" in the  $cm$  frame (Drechsel and Tiator, 1992), and all quantities being expressed in that frame. For an inclusive reaction the cross section has to be summed over the azimuthal angle  $\Phi \equiv \Phi_\pi$ . Due to their definition with regard to the reaction plane, also the components  $P_l, P_t, P_n$  depend on the pion angles,  $(\Theta_\pi, \Phi_\pi)$ . In this way also combinations like  $P_n \sin \Phi$ , etc., give finite contributions to the angular integration. Defining then the inclusive cross sections (in a somewhat symbolic way !) by

$$\sigma_i = \int \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} R_i(\Theta_\pi) d\Omega_\pi, \tag{16}$$

we obtain

$$\sigma^{(v)} = \sigma_T + \varepsilon_L \sigma_L + hP_x \sqrt{2\varepsilon_L(1-\varepsilon)} \sigma_{LT'} + hP_z \sqrt{1-\varepsilon^2} \sigma_{TT'}, \tag{17}$$

i.e. two structure functions ( $L, T$ ) without polarization and two others ( $LT'$  and  $TT'$ ) for a double polarization experiment. Up to kinematical factors, the four partial cross sections may be expressed by the CGLN multipoles (Chew *et al.*, 1957),

$$\sigma_T = 4\pi \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \sum_l \frac{1}{2} (l+1)^2 \left[ (l+2) (|E_{l+}|^2 + |M_{l+1,-}|^2) + l (|M_{l+}|^2 + |E_{l+1,-}|^2) \right], \tag{18}$$

$$\sigma_L = 4\pi \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \sum_l (l+1)^3 \left[ |L_{l+}|^2 + |L_{l+1,-}|^2 \right], \tag{19}$$

$$\sigma_{LT'} = 4\pi \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \sum_l \frac{1}{2} (l+1)^2 \left[ -L_{l+}^* ((l+2)E_{l+} + lM_{l+}) + L_{l+1,-}^* (lE_{l+1,-} + (l+2)M_{l+1,-}) \right], \tag{20}$$

$$\begin{aligned}
\sigma_{TT'} &= 4\pi \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \sum_l \frac{1}{2} (l+1) \left[ -(l+2) (|E_{l+}|^2 + |M_{l+1,-}|^2) + l (|M_{l+}|^2 + |E_{l+1,-}|^2) \right. \\
&\quad \left. - 2l(l+2) (E_{l+}^* M_{l+} - E_{l+1,-}^* M_{l+1,-}) \right]. \tag{21}
\end{aligned}$$

Note that the "unpolarized" functions,  $\sigma_L$  and  $\sigma_T$ , contain only positive contributions, while the "polarized" ones,  $\sigma_{LT'}$  and  $\sigma_{TT'}$ , are arithmetic sums with alternating signs. The Fermi-Watson theorem (Watson, 1954) guarantees that phases of all multipoles with index  $l_+$  or  $l_-$  carry the same phase of the corresponding  $\pi N$  partial wave, i.e.  $\sigma_{LT'}$  contains only a real part. It is interesting to note that the third line in eq. (15) will formally give rise to a cross section  $\sigma_{LT}$ . However, this cross section is precisely the imaginary part

of the multipole combination of  $\sigma_{LT'}$ , i.e. it vanishes in the one-photon exchange approximation, also in the energy region of more-pion and other particle production because of the unitarity for the S matrix.

The leading contributions for the two spin-polarized structure functions are

$$\sigma_{LT'} = 4\pi \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \{-2L_{1+}^*(M_{1+} + 3E_{1+}) + L_{1-}^*M_{1-} - L_{0+}^*E_{0+} + L_{2-}^*E_{2-} \pm\} \quad (22)$$

$$\sigma_{TT'} = 4\pi \frac{|\vec{k}_\pi|}{k_\gamma^{cm}} \{|M_{1+}|^2 - 6E_{1+}^*M_{1+} - 3|E_{1+}|^2 - |M_{1-}|^2 - |E_{0+}|^2 + |E_{2-}|^2\} \quad (23)$$

The bulk contribution to the DHG integrand,  $\sigma_{TT'}$ , comes from the  $\Delta(1232)$  resonance multipoles  $1^+$ ; the higher resonances  $N^*(1440)$ ,  $N^*(1535)$  including the S-wave threshold production, and  $N^*(1520)$  contribute the multipoles  $1^-$ ,  $0^+$  and  $2^-$ , in that order. Finally,  $\sigma_{TT'}$  and  $\sigma_{LT'}$  are related to the Bjorken structure functions (Bjorken, 1966)  $G_1$  and  $G_2$  by

$$\sigma_{LT'} = -\frac{4\pi^2\alpha m}{1-x} \left( G_1(\nu, Q^2) + \frac{\nu}{m} G_2(\nu, Q^2) \right), \quad (24)$$

$$\sigma_{TT'} = -\frac{4\pi^2\alpha m}{1-x} \left( G_1(\nu, Q^2) - 2x G_2(\nu, Q^2) \right), \quad (25)$$

where  $x = Q^2/2m\nu$  is the Bjorken scaling variable. In terms of such structure functions, the inclusive cross section,

$$\frac{d^2\sigma}{d\Omega'_e dk'_e} = \frac{\alpha^2}{Q^4} \frac{k'}{k} L_{\mu\nu}(k) W^{\mu\nu}(p), \quad (26)$$

may be expressed by a leptonic ( $L_{\mu\nu}$ ) and an hadronic tensor ( $W^{\mu\nu}$ ). Both may be decomposed into a symmetric ( $\mathcal{S}$ ) and an antisymmetric ( $\mathcal{A}$ ) part,

$$W_{\mu\nu}^{(\mathcal{S})} = \frac{1}{m} \left( \frac{q_\mu q_\nu}{q^2} - g_{\mu\nu} \right) F_1(\nu, Q^2) + \frac{1}{m\nu} \tilde{p}_\mu \tilde{p}_\nu F_2(\nu, Q^2) \quad (27)$$

$$W_{\mu\nu}^{(\mathcal{A})} = \epsilon_{\mu\nu\alpha\beta} q^\alpha \left( m s^\beta G_1 + \frac{1}{m} (s^\beta p \cdot q - p^\beta s \cdot q) G_2 \right),$$

where  $\tilde{p}^\nu = p^\nu - (p \cdot q/q^2)q^\nu$  is a gauge invariant vector. Except for normalization factors, the leptonic tensor has the same structure as eq. (27), with all form factors equal to 1. Since  $s^\beta \sim p^\beta$  for high energetic leptons, only the first term in  $L_{\mu\nu}^{(\mathcal{A})}$  contributes. Its contraction with  $W_{\mu\nu}^{(\mathcal{A})}$  gives rise to the spin-dependent parts of the cross section depending on the helicity  $h$  of the electron.

The DHG integral, eq. (8), is

$$I(Q^2) = \frac{m^2}{4\pi\alpha} \int \frac{d\nu}{\nu} (1-x) \sigma_{TT'}(\nu, Q^2), \quad (28)$$

and a similar integral,  $J(Q^2)$ , may be obtained for the structure function  $\sigma_{LT'}$ . Note that the integral runs from threshold to infinity. In the scaling region ( $Q^2, \nu \rightarrow \infty; x$  fixed)

the structure functions may be expressed by the quark distribution functions,

$$\begin{aligned} m^2 \nu G_1(\nu, Q^2) &= g_1(x, Q^2) \Rightarrow g_1(x) = \frac{1}{2} \sum_i e_i^2 (f_i^\uparrow(x) - f_i^\downarrow(x)) \\ m \nu^2 G_2(\nu, Q^2) &= g_2(x, Q^2) \Rightarrow g_2(x) = \frac{1}{2} \sum_i e_i^2 (f_i^{\rightarrow}(x) - f_i^{\leftarrow}(x)) - g_1(x), \end{aligned} \quad (29)$$

where  $f_i^{\uparrow, \downarrow}$  and  $f_i^{\rightarrow, \leftarrow}$  denote the densities for longitudinal and transverse quark polarization, respectively. Changing the integration variable from  $\nu$  to  $x$ , the two sum rules may be expressed by the quark spin distributions,

$$I(Q^2) = \frac{2m^2}{Q^2} \int dx \left[ g_1(x, Q^2) - \frac{4x^2 m^2}{Q^2} g_2(x, Q^2) \right], \quad (30)$$

$$J(Q^2) = \frac{2m^2}{Q^2} \int dx [g_1(x, Q^2) + g_2(x, Q^2)] \equiv J_1 + J_2. \quad (31)$$

Since the second term in eq. (30) is small under the usual experimental conditions, we expect

$$J_1(0) \approx I(0) = -\frac{\kappa^2}{4}. \quad (32)$$

As has been recently pointed out by Soffer and Teryaev (1993), the integral  $J_2$  is related to the so-called "Burkhardt-Cottingham sum rule" (Burkhardt and Cottingham, 1970; Heimann, 1973). Further aspects of this sum rule have been discussed in the early 70's (Feynman, 1972; Schwinger, 1975; Tsai *et al.*, 1975). Since  $g_2(\nu, Q^2)$  is an odd function under crossing, ( $\nu \rightarrow -\nu$ ), its sum over all intermediate states vanishes. As a consequence the contribution over the excited states is exactly cancelled by the ground state expectation value, leading to

$$J_2(Q^2) = \frac{\mu}{4} G_M(Q^2) (\mu G_M(Q^2) - G_E(Q^2)), \quad (33)$$

where  $\mu$  is the total magnetic moment, and  $G_E$  and  $G_M$  are the electric and magnetic form factors of the nucleon. In particular

$$J_2(0) = \frac{1}{4} \kappa \mu, \quad J(0) = \frac{1}{4} \kappa (\mu - \kappa). \quad (34)$$

## THEORIES AND MODELS

### Experimental Status

The present "experimental" situation for the DGH sum rule for the proton ( $I_p$ ) and neutron ( $I_n$ ) is summarized in Figs. 6 and 7, respectively, by the solid line labeled "phenomenological model" (Burkert *et al.*, 1991; Kuhn *et al.*, 1993). It is obtained by fitting a set of resonances, based on a relativistic quark model, to the data. At  $Q^2 = 0$  it agrees reasonably well with the previous analysis of pion photoproduction (Karliner,



1973) and a later analysis by the Virginia group (Workman and Arndt, 1992). The clear disagreement of these results with the DHG prediction for the neutron, as seen in Fig. 7, is certainly a good motivation to repeat the experiment. Another striking feature is the rapid decrease from the large absolute values at small  $Q^2$  to values around zero at  $Q^2 \approx 1\text{GeV}^2$ . Finally, for  $Q^2 \geq 2\text{GeV}^2$ , the DHG integral should have its asymptotic  $Q^{-2}$  behaviour with a constant determined by the EMC/SLAC experiments of deep inelastic scattering (DIS). The error bars given in the two figures indicate the projected range and accuracy of the planned CEBAF experiments.

### Vector Meson Dominance (VMD)

A global fit to the data has been given in a model inspired by VMD (Anselmino *et al.*, 1989),

$$I(Q^2) = \left( -\frac{\kappa^2}{4} + \frac{\mathcal{Z}Q^2m^2}{m_V^2} \right) \bigg/ \left( 1 + \frac{Q^2}{m_V^2} \right)^2, \quad (35)$$

where  $m_V$  is the mass of the vector mesons and  $\mathcal{Z}$  has been determined by DIS. It describes both the behaviour at small and large  $Q^2$  and predicts a sign change at  $Q^2 \approx m_V^2$ . In a somewhat different parametrization, Burkert and Ioffe (1992) have fitted the sum rule to the  $\Delta(1232)$  contribution plus monopole and dipole forms.

### Constituent Quark Model (CQM)

As may be seen in Fig. 6, the quark model (even in its "relativized" versions !) fails in describing the DHG. This is very surprising, indeed, because the model gives a good overall description of the excitation spectrum of the nucleon (Isgur and Karl, 1978 and 1979). In the following we will demonstrate the reasons for this blatant failure for the case of its nonrelativistic version. In its simplest version the model has a quark mass  $m_q = m/3$ , an oscillator parameter related to the size of the object,  $\alpha_0^{-2} = \langle r^2 \rangle$ , and Dirac point particles leading to  $\kappa = 2$ . Including the usual hyperfine interaction and a configuration mixing of  $0\hbar\omega$  and  $2\hbar\omega$  states, the wave function of the nucleon is

$$|N\rangle = a_S|^2S_{1/2}\rangle_S + a_{S'}|^2S'_{1/2}\rangle_S + a_M|^2S'_{1/2}\rangle_M + a_D|^4D_{1/2}\rangle_M, \quad (36)$$

with admixture coefficients  $a_S = 0.93$ ,  $a_{S'} = -0.29$ ,  $a_M = -0.23$ , and  $a_D = -0.04$  (Giannini, 1991).

The corresponding strength of the hyperfine interaction has been obtained by fitting the positions of the first and second resonance region. The final result for the DHG integral is (Drechsel and Giannini, 1993; DeSanctis *et al.*, 1994)

$$I(Q^2 = 0) = -1 + 2a_M^2 + \frac{5}{2}a_D^2 \pm \frac{1}{2}a_D^2 + [a^4], \quad (37)$$

the upper and lower sign corresponding to proton and neutron, respectively. A comparison with eq. (13) gives  $I_{VV} = -0.86$ , in excellent agreement with experiment, and  $I_{SV}$  with the proper sign but too small in magnitude. The isoscalar magnetic moment cannot be explained by the small  $D$ -state admixture, but probably requires an introduction of sea quark effects as in the case of the Ellis-Jaffe sum rule (Ellis and Jaffe, 1974). In order to obtain the result of eq. (37) independently from both the integral (sum over the

excited states) and the ground state value of the magnetic moment, the calculation has to be performed very "carefully", however. In fact, the complete calculation without the hyperfine interaction gives

$$I = -1 + \frac{5}{4} \frac{1 + \tau_0}{2} \sum_{n \geq 1} \frac{1}{n!} (n\zeta)^{2n} e^{-(n\zeta)^2}, \quad (38)$$

where the sum is over all oscillator shells ( $n \geq 1$ ), and the expansion parameter is

$$\zeta^2 = (\hbar\omega_0/\sqrt{3}\alpha_0)^2 = 1/(3m_q^2 < r^2 >) \approx 0.57, \quad (39)$$

with a value for  $\alpha_0$  to describe the helicity structure of the spectrum (Copley *et al.*, 1969). Clearly the higher order terms are retardation terms of higher order in  $(\omega/m_q)^2$  than can be described in a nonrelativistic model. The correct result can only be obtained for the leading order term, i.e. by neglecting all terms of order  $m_q^{-2}$  or, alternatively, by replacing the  $\omega^2$ -dependence of the retardation terms by a relativistic  $Q^2$ -dependence.

Even if we neglect higher order retardation, a further inconsistency appears if the hyperfine interaction is switched on. The reason has been pointed out long ago (Brodsky and Primack, 1969; Close and Copley, 1970; Krajcik and Foldy, 1974; DeSanctis and Prospero, 1987). In order to fulfill the algebra of the Poincaré group (translations, rotations and boosts) for an interacting many-body system, two-body currents are required,

$$\vec{J} = \sum_k \left( \frac{\vec{p}_k}{m_q} + i \frac{\vec{\sigma}_k \times \vec{q}}{2m_q} \right) e^{i\vec{q} \cdot \vec{r}_k} + \vec{J}_{rel}(1\text{-body}) + \vec{J}_{rel}(2\text{-body}). \quad (40)$$

It is not sufficient to include the relativistic spin-orbit current and other corrections of order  $m_q^{-2}$  as in the usual "relativized" versions of the CQM. Instead, genuine two-body currents of order  $m^{-1}m_q^{-1}$  appear at the same level, in particular a modification of the electric dipole current due to  $cm$  correlations of the relativistic system. Being functions of the properties of both the struck particle and the total system (total charge, mass and momentum) they are somewhat difficult to treat and, certainly, have been ignored within the framework of single particle transitions.

With the current operator (40) and the given spectrum of the CQM, the sum rule is

$$I \sim \sum_f \frac{1}{\omega^2} |\langle f | J_+ | i \rangle|_{\mathcal{A}-\mathcal{P}}^2, \quad (41)$$

where  $\mathcal{A} - \mathcal{P}$  denotes the difference of the matrix elements for antiparallel spins (initial state nucleon:  $-1/2$ , photon:  $+1$ ) and parallel spins ( $+1/2, +1$ ). Eq. (41) can be expressed in terms of a vector product of the current operators,

$$I \sim \sum_f \frac{1}{\omega^2} \left[ \langle i | \vec{J}^+ | f \rangle \times \langle f | \vec{J} | i \rangle \right]_z, \quad (42)$$

the excitation energy being a function of the states,  $\omega = \omega_{fi}$ . Apparently the leading order convection current does not contribute, because the  $\vec{p} \times \vec{p}$  contributions vanish identically.

The DHG is saturated by the spin current ( $\sim \omega^2 \vec{\sigma} \times \vec{\sigma}$ ), corresponding retardation terms in the orbital angular momentum ( $\sim \omega^2 \vec{l} \times \vec{l}$ ) and relativistic corrections of both one-body and two-body structure of order  $\omega^2$ . Neglecting higher order terms  $O(\omega^4)$ , the  $\omega$ -dependence in eq. (42) cancels and the DHG integral may be evaluated by closure.

Such an evaluation by closure can also be obtained for the individual multipole contributions (Drechsel and Giannini, 1993; DeSanctis *et al.*, 1994). The main results are

- the convection current cancels to leading order,
- the remaining contribution of the  $E_{0+}$ -multipole is cancelled by the complex of  $E_{2-}/M_{2-}$  excitations,
- up to relativistic corrections and small contributions of the hyperfine force, only the (unretarded) spin-part of the  $M_{1+}$  survives,
- the contributions of  $4\hbar\omega_0$ - states to  $I_{SV}$  seem to be large,
- good agreement is reached for  $I_{VV}$ , while the predicted value of  $I_{SV}$  is much too small.

Along these lines we obtain a phenomenological prediction for  $Q^2 > 0$  by replacing

$$J_+ = \frac{\omega\sigma_+}{2m_q} e^{i\vec{q}\cdot\vec{r}} \Rightarrow \frac{\omega\sigma_+}{2m_q} \left(1 - \frac{Q^2 < r^2 >}{6}\right) + \frac{\sigma_+}{2m_q} \frac{Q^2}{\omega + |\vec{q}|}. \quad (43)$$

The first term on the *rhs* shows the unretarded spin current multiplied by a typical form factor, leading to a decrease in absolute value with increasing  $Q^2$ . It is superimposed with the second term, carrying the same sign as for the case of  $Q^2 = 0$ . As a result the slope of the sum rule at  $Q^2 = 0$  could be both positive or negative, depending on the form factor.

### Chiral Perturbation Theory (ChPT)

The discussion of the slope of the DHG integral has been reactivated by a recent calculation in ChPT (Bernard *et al.*, 1993). While a calculation of the integral itself, being of order  $\kappa^2$ , would require at least a two-loop calculation, its derivative has been obtained both in the framework of relativistic ChPT and within the heavy baryon approximation. The result is shown in Fig. 8. Obviously the difference between the two predictions is large, and both differ from the result of the "phenomenological" prediction (Fig. 6). The wide range of the theoretical predictions is connected with the bad convergence of the loop expansion in the case of the nucleon. Contrary to pionic problems, where all energies, momenta and masses are small near threshold, the large mass of the nucleon sets an additional (large !) scale. As a consequence, the explicit  $1/m$  expansion of the heavy baryon formalism converges much faster and, probably, leads to a better prediction. As shown in the previous subsection, however, relativistic corrections play an important role in the case of the DHG, and it remains to be seen whether the leading term in the heavy mass formulation of the ChPT is really sufficient. Though  $\Delta$ -loops do not play a major role at the one-loop level, it is questionable whether such resonance phenomena can be appropriately described to that order.

## Current Algebra and large $Q^2$

At higher values of  $Q^2$  an expansion of the current in  $m_q^{-2}$  does not make sense. Instead one has to use the relativistic current operator. A "back of an envelope" calculation gives for the relevant component of the current, in the Breit frame of the struck parton,

$$J_+ = \sum_k \frac{e_k |\vec{q}|}{2m_q} \sigma_+(k), \quad (44)$$

where  $Q^2 = \vec{q}^2$  in the Breit system. The DHG integral, eq. (41), becomes

$$I \sim \sum_f \sum_k \frac{1}{\nu^2} \left| \left\langle f \left| \frac{e_k |\vec{q}|}{2m_q} \sigma_+(k) \right| i \right\rangle \right|_{\mathcal{A}-\mathcal{P}}^2. \quad (45)$$

Rewriting this expression in terms of the Bjorken scaling variable  $x$ , we obtain

$$I \sim \frac{1}{Q^2} \sum_k \sum_f \left| \left\langle f \left| e_k \frac{m}{m_q} x \sigma_+(k) \right| i \right\rangle \right|_{\mathcal{A}-\mathcal{P}}^2. \quad (46)$$

In the naive parton model we find  $x = m_q/m \approx 1/3$ . Hence the sum over the final states may be performed, and

$$I(Q^2) = \frac{2m^2}{Q^2} \Gamma_1, \quad \Gamma_1 = \frac{1}{6} \cdot \frac{5}{3} \cdot \frac{1 + \tau_0}{2}. \quad (47)$$

The factor  $g_A/g_V = 5/3$  is the prediction of the simple quark model for the axial coupling constant, and  $\Gamma$  is related to the quark spin distribution by

$$I(Q^2) \Rightarrow \frac{2m^2}{Q^2} \int_0^1 dx g_1(x) \equiv \frac{2m^2}{Q^2} \Gamma_1. \quad (48)$$

We note in passing that

$$\Gamma_1^p - \Gamma_1^n = \frac{1}{6} \frac{g_A}{g_V} (1 - \alpha_s(Q^2) \pm \dots) \approx 0.19 \quad (49)$$

is given by current algebra (Bjorken, 1966), while the individual values of  $\Gamma^p$  and  $\Gamma^n$  become model dependent (Ellis and Jaffe, 1974). For a more detailed discussion see the contribution by B. Frois (1994). As has been pointed out in sect. 3, the Burkhardt-Cottingham sum rule implies  $\Gamma_2^p = \Gamma_2^n = 0$  for  $Q^2 \rightarrow \infty$ .

Let us finally comment on the role of current algebra for the DHG sum rule. The asymptotic limit of the sum rule at large  $Q^2$  was first discussed by Bjorken (1966) on the basis of the equal-time current commutator

$$[j_\mu(0, \vec{r}), j_\nu(0, 0)] = -2i\epsilon_{\mu\nu\lambda\rho} q^\lambda j_5^\rho \delta(\vec{r}) + \text{gradients}. \quad (50)$$

Comparing with our eq. (42), we immediately find that the vector product in that equation is nothing else than the commutator of the space-like parts of the current, hence  $(\vec{J} \times \vec{J})_z \sim (\vec{J}_5)_z = g_A \bar{\psi} \gamma_z \gamma_5 \psi$ . As a result the DHG integral in the scaling region is given by the axial current, in particular  $I^p - I^n \sim g_A^{(3)} \approx 5/4$ . In view of the experimental evidence in 1966, Bjorken was not too much impressed with the possible consequences of his work. He wrote: "Something has to be salvaged from this worthless equation by constructing an inequality...", and derived an upper limit for the spin-averaged total cross section.

### QCD Sum Rules

As an example of higher-twist calculations extrapolating Bjorken's result to smaller  $Q^2$ , we refer to a recent QCD based prediction of Balitsky *et al.* (1990). They find only small corrections to the asymptotic behaviour,

$$\Gamma^p - \Gamma^n \approx \frac{1}{6} \left( (1 - \alpha_s(Q^2)) g_A - \frac{0.3 \text{GeV}^2}{Q^2} + \left[ \frac{1}{Q^4} \right] \right). \quad (51)$$

Even at the  $Q^2 = 1 \text{GeV}^2$ , the smallest reasonable value for such an expansion, the correction is only 25%. Hence the DHG integral at the real photon point should be saturated by contributions dying out faster than  $1/Q^4$  in the asymptotic limit.

## THE SPIN STRUCTURE OF RESONANCES

The integrand of the DHG sum rule is determined by the helicity structure of the integrated cross section. In the resonance region these contributions may be decomposed either in electric and magnetic multipoles or in "helicity amplitudes",

$$A_{1/2} = \sqrt{\frac{4\pi\alpha}{2k_\gamma^{cm}}} \langle N^*(J', M' = +\frac{1}{2}) | J_+ | N^*(J = \frac{1}{2}, M = -\frac{1}{2}) \rangle \quad (52)$$

$$A_{3/2} = \sqrt{\frac{4\pi\alpha}{2k_\gamma^{cm}}} \langle N^*(J', M' = +\frac{3}{2}) | J_+ | N^*(J = \frac{1}{2}, M = +\frac{1}{2}) \rangle \quad (53)$$

$$S_{1/2} = \sqrt{\frac{4\pi\alpha}{2k_\gamma^{cm}}} \langle N^*(J', M' = +\frac{1}{2}) | J_+ | N^*(J = \frac{1}{2}, M = +\frac{1}{2}) \rangle, \quad (54)$$

the latter describing the longitudinal current. In the case of the  $A_{\frac{3}{2}}$  amplitude, the photon can be absorbed by a single quark without a helicity flip, while the  $A_{\frac{1}{2}}$  amplitude requires quark spin flips (see Fig. 9). Since the quark masses can be neglected in the limit of large momentum transfer,  $A_{\frac{1}{2}} \sim Q^{-3}$  becomes the dominant amplitude in that limit, and  $A_{\frac{3}{2}} \sim Q^{-5}$  should be strongly suppressed (LePage and Brodsky, 1980)

### The First Resonance Region

This region between threshold and about  $400 \text{MeV}$  excitation energy is dominated by the  $P_{33}$  (1232) or  $\Delta(3,3)$  resonance, clearly visible in Fig. 3 on top of a broad background of

mostly S-wave pions. Within the harmonic oscillator quark model, the  $\Delta$  and the nucleon are partners with configuration  $\{56, 0^+\}_0$ , i.e. members of the symmetrical 56-plet of  $SU(6)$ , orbital momentum  $L = 0$ , positive parity and no radial nodes. In this approximation the  $\Delta$  may only be excited by the magnetic dipole ( $M1$  or  $M_{1+}$ , respectively). As has been stated previously, the introduction of a hyperfine interaction leads to an admixture of mixed symmetry states of the 70-plet in connection with orbital or radial excitation. Of particular significance is the admixture of a  $D$ -state component leading to the existence of a small electric quadrupole transition ( $E2$  or  $E_{1+}$ , respectively). The helicity amplitudes for this resonance are superpositions of the corresponding multipoles,

$$A_{\frac{1}{2}} = -\frac{1}{2}(M1 + 3 \cdot E2), \quad A_{\frac{3}{2}} = -\frac{\sqrt{3}}{2}(M1 - E2). \quad (55)$$

Without hyperfine interactions, these amplitudes are simply proportional,  $A_{\frac{3}{2}} = \sqrt{3}A_{\frac{1}{2}}$ . On the other hand, perturbative  $QCD$  predicts that the spin-flip amplitude  $A_{\frac{3}{2}}$  should vanish. Therefore the ratio  $EMR \equiv E2/M1$  should approach unity in the limit  $Q^2 \rightarrow \infty$ . In the low-energy regime, however, the  $D$ -state probability of both nucleon and  $\Delta$  is of the order of 1 %, leading only to a small quadrupole moment of the  $\Delta$ ,  $Q_\Delta \approx -0.089 fm^2$ .

Careful studies have shown that the polarized photon asymmetry  $\Sigma$  is the most sensitive observable for experiments with real photons (Blanpied *et al.*, 1992). Many more choices seem to exist for electroexcitation with polarization degrees of freedom, apparently some of the longitudinal and transverse interference terms are very sensitive to both the  $E_{1+}$  and  $L_{1+}$  amplitudes. The coincidence  $\vec{e} + p \rightarrow e' + \vec{p}' + \pi^0$ , with polarization transfer to the proton, is a particularly well suited experiment (Lourie, 1990; Hanstein, 1993). The present value is  $EMR \approx -1.5\%$  at the real photon point with some indications that it becomes slightly positive at  $Q^2 \approx 3 GeV^2$  (see Fig. 10). The corresponding ratio  $SMR \equiv S_{1+}/M_{1+}$  is also negative with large error bars and partially contradicting experimental evidence. The recent Bonn data (Kalleicher, 1993) indicate a relatively strong fluctuation as function of  $Q^2$ . The measured value at  $Q^2 \approx 0.1 GeV^2$ ,  $SMR \approx -13\%$ , corresponds to  $EMR \approx -6\%$ .

### The Roper Resonance $P_{11}(1440)$

In the  $CQM$  the Roper is a radial excitation of the nucleon occuring at an energy of  $2\hbar\omega_0$ . In units of  $10^3/GeV^{\frac{1}{2}}$ , the measured helicity amplitude for the proton is  $A_{\frac{1}{2}}^p = -70 \pm 5$ , the value for the neutron is  $23 \leq A_{\frac{1}{2}}^n \leq 56$ . The  $CQM$  predicts a ratio  $A_{\frac{1}{2}}^n/A_{\frac{1}{2}}^p = -\frac{2}{3}$ , in reasonable agreement with the data within the large error bars. However, the values for the  $CQM$  amplitudes themselves are too small by a factor of 3. The chiral bag model ( $CBM$ ) predicts a ratio of  $-1$  for the pionic contributions. With decreasing bag radius  $r_0$ , these effects of the pion cloud increase strongly, e.g.  $A_{\frac{1}{2}}^p = -36, -80$  and  $-147$  for  $r_0 = 1 fm, 0.8 fm$  and  $0.6 fm$ , respectively (Drechsel, 1994).

As has been pointed out by Li *et al.* (1992), explicit gluon degrees of freedom might play a role even at low excitation energies. The wave function of such a "hybrid" contains two components,

$$|N^*\rangle = \alpha |q^3\rangle + \sqrt{1 - \alpha^2} |q^3 \times g\rangle. \quad (56)$$

The gluon appearing in the second term requires a quark configuration  $q^3$  with colour in order to insure an overall colour neutral wave function. As a consequence the quarks can now be in the configuration  $[70, 0^+]_0$ , i.e. with mixed symmetry in  $SU(6)$  classification and neither orbital nor radial nodes. In this case the wave function in  $\vec{r}$ -space may be identical to that one of the nucleon, leading to

$$A_{\frac{1}{2}}(q^3 g)/A_{\frac{1}{2}}(q^3) \sim \frac{1}{Q^2}. \quad (57)$$

The corresponding Coulomb amplitude vanishes except for relativistic corrections,  $S_{\frac{1}{2}}(q^3 g) \approx 0$ , because the longitudinal photon cannot excite the transverse colourmagnetic field of the gluon. The Roper is certainly a good candidate for such a "hybrid", because it occurs at an extremely low energy for a  $2\hbar\omega_0$  state of the  $CQM$ . Up to now the Roper has not been seen very clearly in electromagnetic reactions. The size of its Coulomb excitation  $S_{\frac{1}{2}}$  will be quite essential for its classification. While a small or vanishing value will be an indication of a hybrid, very large contributions should be typical of explicit pion degrees of freedom as predicted by the  $CBM$ . The present status of the data on the Roper is compared to various predictions in Fig. 11.

It is also interesting to note that a broad bump has been seen near the Roper resonance in a missing energy spectrum for  $\alpha - p$  scattering, which could be an indication for a strong monopole transition (Morsch *et al.*, 1992).

### The Second and Third Resonance Region

The second resonance region contains the two dipole excitations  $S_{11}(1535)$  and  $D_{13}(1520)$  with spins  $\frac{1}{2}^-$  and  $\frac{3}{2}^-$ , respectively. In the  $CQM$  its configurations are  $\{70_M, 1_{MJ}^-\}_1$ . The most prominent state in the third resonance region is the  $F_{15}(1680)$  with configuration  $\{56_S, 2^+\}_2$ . These states have quite different properties as function of momentum transfer. In the  $CQM$  we have

$$\begin{aligned} A_{\frac{1}{2}}(S_{11}) &\sim \left( \frac{\vec{q}^2}{\alpha^2} + 2 \right) F(\vec{q}^2), \\ A_{\frac{1}{2}}(D_{13}) &\sim \left( \frac{\vec{q}^2}{\alpha^2} - 1 \right) F(\vec{q}^2), \\ A_{\frac{3}{2}}(S_{11}) &\sim F(\vec{q}^2), \\ A_{\frac{1}{2}}(F_{15}) &\sim \left( \frac{\vec{q}^2}{\alpha^2} - 2 \right) |\vec{q}| F(\vec{q}^2), \\ A_{\frac{3}{2}}(F_{15}) &\sim |\vec{q}| F(\vec{q}^2). \end{aligned} \quad (58)$$

The two contributions to the  $A_{\frac{1}{2}}$  amplitudes are due to the spin and orbital currents of the quark motion. At the real photon point,  $\vec{q}^2 = \omega^2$ , the experiments indicate a cancellation of the two currents in the case of the proton,  $A_{\frac{1}{2}}^p(D_{13}) \approx 0 \approx A_{\frac{1}{2}}^p(F_{15})$ . Using  $\alpha \sim 0.17 GeV^2$ , this cancellation is nearly complete for both resonances, which may be considered as one of the early successes of the  $CQM$  (Copley *et al.*, 1969). Replacing

$\vec{q}^2 \rightarrow Q^2$  (i.e. performing the nonrelativistic calculations in the Breit frame), we find that the amplitudes  $A_{\frac{1}{2}}$  become increasingly important for large  $Q^2$ . This is in agreement with *PQCD*, requiring the dominance of the helicity conserving amplitudes  $A_{\frac{1}{2}}$  in the asymptotic region. Within *PQCD* the prediction is  $A_{\frac{1}{2}} \sim Q^{-3}$  and  $A_{\frac{3}{2}} \sim Q^{-5}$  for  $Q^2 \rightarrow \infty$ . The rapid change at  $Q^2 \approx 0.5 \text{ GeV}^2$  is reflected most clearly by the helicity asymmetry shown in Fig. 12. The value  $(A_{\frac{1}{2}} - A_{\frac{3}{2}})/(A_{\frac{1}{2}} + A_{\frac{3}{2}})$  ranges between the lowest possible ratio -1 at the real photon point and the highest possible ratio +1 for  $Q^2 \rightarrow \infty$ , for the strongest states of both the second ( $D_{13}$ ) and third ( $F_{15}$ ) resonance region.

### Eta Production

In comparison with its partner  $D_{13}(1520)$ , the dipole excitation  $S_{11}(1535)$  is only weakly seen in pion photoproduction. However, it couples very strongly to the  $\eta$  meson, about 50% of its decay width is due to  $\eta$  emission. In comparison, the  $D_{13}$  has only a  $10^{-3}$  branch for  $\eta$  decay, and also an excited  $S_{11}$  occurring in the third resonance region couples only weakly to the  $\eta$ . The only other resonance with a sizeable  $\eta$  branch is the  $P_{11}(1710)$  with 25%  $\eta$  decay. Though the overall contribution of the  $\eta$  to sum rules will be small, the study of this decay channel is interesting because of its connection with strangeness degrees of freedom. The data seem to indicate a rather slow decrease of the transition form factor to the  $S_{11}$  resonance as function of  $Q^2$ .

## PERSPECTIVES AND CONCLUSIONS

Investigations with electromagnetic interactions have contributed substantially to a better understanding of the structure of hadrons. However, previous experiments have been limited by small currents and low duty-factors. As a consequence the statistics for small amplitudes has been bad and the signal to noise ratio has been small. With the advent of the new electron accelerators new classes of coincidence experiments have become possible, and polarization degrees of freedom will play an important role. With a beam polarization of 40% and more, polarized electrons promise to provide a new capability to measure some of the most wanted observables, in particular in combination with target and recoil polarization. In the nucleon resonance region such systematic investigations with complete kinematics and separation of the independent structure functions include:

- a model independent determination of the quadrupole amplitudes  $E_{1+}$  and  $L_{1+}$  in the region of the  $\Delta$  resonance ("bag deformation"),
- the measurement of the monopole strength  $L_{1-}$  near the Roper resonance ("breathing mode vs. hybrid"),
- the analysis of the helicity asymmetry of the nucleon resonances with its strong dependence on momentum transfer,
- the "tagging" of the weak  $S_{11}$  dipole resonance by the  $\eta$  channel and, by precision experiments, the coupling of the  $\eta$  to other resonances as function of momentum



transfer  $Q^2$ .

The helicity structure of the photo- and electroproduction cross sections is related to the spin structure of the nucleon in deep inelastic lepton scattering. Both the generalized Drell-Hearn-Gerasimov sum rule and the Burkhardt-Cottingham sum rule define energy-weighted integrals over the excitation spectrum from the photonuclear point ( $Q^2 = 0$ ) to asymptotic values of momentum transfer, where the experiment probes the spin distribution function. Since these sum rules have been derived on the basis of quite general principles (relativity, causality, unitarity, gauge invariance), they provide a unique testing ground for our understanding of the nucleon. In particular, the sum rules connect ground state properties (magnetic moments and form factors) with the helicity structure of the excitation spectrum.

Up to now neither of these sum rules has been tested by a direct experiment. There is still the possibility that the sum rules will not converge. Such a failure would indicate that even the ground state properties of the nucleon are determined by phenomena happening at asymptotically large energies, a situation which would send all model-builders back to the drawing board. A series of experiments is underway to clarify the situation. In a collaboration of Bonn and Mainz groups (Arends *et al.*, 1993), the spin structure in the resonance region will be studied with real photons to find out whether the DHG sum rule converges and, ultimately, whether the proton-neutron difference is an indication of a possible breakdown of our theoretical concepts.

In the region of the order of  $Q^2 \sim 1 - 2 \text{ GeV}^2$ , various CEBAF experiments (Burkert *et al.*, 1991; Kuhn *et al.*, 1993) will explore the sum rules in the transition region from coherent resonance excitation to deep inelastic scattering. Of particular interest will be the rapid crossover of the DHG integral from large negative to positive values and the question whether the predictions of the Burkhardt-Cottingham sum rule can be established. All of these experiments will require a high degree of precision and a careful analysis of the systematic errors. However, they will help to increase our knowledge of hadronic structure in a truly qualitative way and provide a good chance to discover new and exiting phenomena.

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## FIGURES

Fig. 1: Real or virtual Compton scattering off the nucleon. The four-momenta of photon and nucleon in the initial state are denoted by  $q = (\omega, \vec{q})$  and  $p = (E, \vec{p})$ , respectively, with an additional "prime" for the final states. The photon polarizations are  $\hat{e}$  and  $\hat{e}'$ , and the nucleon has charge  $e$ , mass  $m$ , and anomalous magnetic moment  $\kappa$ . Note:  $\omega_{lab} = \nu$ .

Fig. 2: Measurement of the DHG sum rule. Left: the spins of photon and nucleon are parallel, the projection of total angular momentum is  $J_z = \frac{3}{2}$ . Right: antiparallel spins,  $J_z = \frac{1}{2}$ .

Fig. 3: The total photoabsorption cross section  $\sigma_T$  for the proton in the resonance region as function of the photon energy  $E_\gamma = \nu$ . Also shown are the main decay channels.

Fig. 4: The difference of the photoabsorption cross sections for the two helicities  $\sigma_{3/2} - \sigma_{1/2}$  as function of  $\nu$ , in the resonance region (Karliner, 1973). From left to right: isovector (VV), isovector-isoscalar interference (SV), and isoscalar (SS) contributions. Note the difference in scale !

Fig. 5: Kinematics for double-polarization experiments. Left: The incoming electron with helicity  $h$  is scattered off a nucleon target with polarization  $\vec{P}$ . The latter is analyzed in a frame with axes  $x$  and  $z$  in the electron scattering plane, and  $y$  perpendicular to the plane. Right: The recoil polarization of the nucleon is analyzed in the reaction plane of the final-state hadrons, e.g. proton and pion. Its axes are  $\vec{l}$  (along the direction of the nucleon),  $\vec{t}$  (sideways, or transverse in the reaction plane), and  $\vec{n}$  (perpendicular to the reaction plane).

Fig. 6: The DHG integral for the proton,  $I_p$ , as function of  $Q^2$  compared to different models. The full and dash-dotted line are phenomenological models with different assumptions on the Roper resonance, the dashed and double-dotted line is the vector dominance model, the two dashed lines starting near the origin are the predictions of (relativized) quark models. The dashed line at positive values indicates the data of EMC/SLAC experiments, the error bars have the predicted accuracy of the planned CEBAF experiment (Burkert *et al.*,

Fig. 7: The DHG integral for the neutron,  $I_n$ , as function of  $Q^2$  compared to different models. The full curve is based on an analysis of pion photoproduction in the resonance region, the dotted and dashed curves are for different assumptions on the Roper resonance (nonrelativistic quark model and hybrid state containing explicit gluon degrees of freedom), the curve labeled ChPT has the slope predicted by Bernard *et al.* (1993). The error bars have the predicted accuracy of the planned CEBAF experiment (Kuhn *et al.*, 1993).

Fig. 8: Momentum dependence ( $-k^2 = Q^2$ ) of the extended DHG sum rule,  $\tilde{I}_p(Q^2) = I_p(Q^2) - I_p(0)$ . The solid line gives the one-loop result in the heavy baryon limit of ChPT, the dashed line includes additional one-loop graphs with  $\Delta(1232)$  resonances, and the dot-dashed curve is the result of the relativistic one-loop version of ChPT (Bernard *et al.*, 1993).

Fig. 9: The "helicity amplitudes" for electroproduction of nucleon resonances for collinear reactions.  $A_{\frac{1}{2}}$ : Incoming nucleon with spin projection  $m = -\frac{1}{2}$  (positive helicity) absorbs a photon with spin  $\lambda = +1$ , leading to  $m' = \frac{1}{2}$  (same helicity as in initial state).  $A_{\frac{3}{2}}$ : For initial spin  $m = +\frac{1}{2}$  (negative helicity) and a photon with  $\lambda = +1$ , the final spin is  $m' = \frac{3}{2}$  (positive helicity, helicity change).  $S_{\frac{1}{2}}$ : For longitudinal photons ( $\lambda = 0$ ) the conservation of spin requires a helicity change.

Fig. 10: The ratio of the electric to magnetic multipole strength for the  $\Delta(1232)$  resonance as function of momentum transfer. Left:  $|E_{1+}/M_{1+}|$ , data from (Burkert, 1990). Right:  $|S_{1+}/M_{1+}|$ . Note that the new data point from Bonn ( $\bullet$ ) at  $Q^2 \approx .12 \text{ GeV}^2$  translates into a value of about - 6% for the ratio  $|L_{1+}/M_{1+}|$  (Kalleicher, 1993).

Fig. 11: The amplitudes  $A_{\frac{1}{2}}$  and  $S_{\frac{1}{2}}$  as function of  $Q^2$  for the Roper resonance. Left: Long-dashed line for  $q^3$  model, solid line  $q^3g$  state, other lines various data analyses. Right: solid line  $q^3$  model, vanishing amplitude for  $g^3g$  state, other lines various data analyses (Li *et al.*, 1992).

Fig. 12: The helicity asymmetry  $(A_{\frac{3}{2}} - A_{\frac{1}{2}})/(A_{\frac{3}{2}} + A_{\frac{1}{2}})$  for electroexcitation of the  $D_{13}(1520)$  and the  $F_{15}(1680)$  as function of  $Q^2$ . The data have been compared to various quark model calculations (Burkert, 1990).

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